Maths (Standard) Delhi (Set 3)

General Instructions :

(i) This question paper comprises four sections – A, B, C and D. This question paper carries 40 questions. All questions are compulsory:
(ii) Section A : Q. No. 1 to 20 comprises of 20 questions of one mark each.
(iii) Section B : Q. No. 21 to 26 comprises of 6 questions of two marks each.
(iv) Section C: Q. No. 27 to 34 comprises of 8 questions of three marks each.
(v) Section D : Q. No. 35 to 40 comprises of 6 questions of four marks each.
(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of one mark each, 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to **attempt only one of the choices** in such questions.
(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
(viii) Use of calculators is not permitted

Question: 1

The value of *k* for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is
(a) −2
(b) ≠2
(c) 3
(d) 2

Solution:

For a system of a quadratic equation to have no solution, the condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.
Given equations are $x + y - 4 = 0 \text{ and } 2x + ky - 3 = 0$, where
$a_1 = 1,\ b_1 = 1,\ c_1 = -4,\ a_2 = 2,\ b_2 = k,\ c_2 = -3.$
We have,
$\frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3}$
Now,
$\frac{1}{2} = \frac{1}{k}$
$\Rightarrow k = 2$
Hence, the correct answer is option (d).

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Question: 2

The HCF and the LCM of 12, 21, 15 respectively are
(a) 3, 140
(b) 12, 420
(c) 3, 420
(d) 420, 3

Solution:

Here,
$12 = 2^2 \times 3$
$21 = 3 \times 7$
$15 = 3 \times 5$
Therefore, HCF(12, 21, 15) = 3 and
LCM(12, 21, 15) = $2^2 \times 3 \times 5 \times 7 = 420$

Hence, the correct answer is option C.

Question: 3

The value of x for which $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an AP, is
(a) 6
(b) −6
(c) 18
(d) −18

Solution:

Given $2x,\ x+10,\ 3x+2$ are the consecutive terms of an AP.
Therefore, the common difference will be same.
$\Rightarrow (x+10) - 2x = (3x+2) - (x+10)$
$\Rightarrow x + 10 - 2x = 3x + 2 - x - 10$
$\Rightarrow 10 - x = 2x - 8$
$\Rightarrow 3x = 18$
$\Rightarrow x = 6$
Hence, the correct answer is option (a).

Question: 4

The first term of an AP is p and the common difference is q, then its 10^{th} term is
(a) $q + 9p$
(b) $p - 9p$

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(c) $p + 9q$
(d) $2p + 9q$

Solution:

The n^{th} term of an AP = a + (n − 1)d, where a and d are the first term and common difference respectively.

Therefore, 10^{th} term = p + (10 − 1)q = p + 9q.

Question: 5

If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is
(a) 10
(b) −10
(c) −7
(d) −2

Solution:

Let the given polynomial be $p(x) = x^2 + 3x + k$
Since, one of the zeroes is 2.
Therefore, the value of $p(x)$ at $x = 2$ will be zero.
Therefore,

$$2^2 + 3 \times 2 + k = 0$$
$$\Rightarrow 4 + 6 + k = 0$$
$$\Rightarrow 10 + k = 0$$
$$\Rightarrow k = -10$$

Hence, the correct answer is option (b).

Question: 6

The total number of factors of a prime number is
(a) 1
(b) 0
(c) 2
(d) 3

Solution:

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The factors of a prime number are 1 and the number itself.
Therefore, the total number of factors of a prime number is 2.

Hence, the correct answer is option (c).

Question: 7

The quadratic polynomial, the sum of whose zeroes is −5 and their product is 6, is
(a) $x^2 + 5x + 6$
(b) $x^2 - 5x + 6$
(c) $x^2 - 5x - 6$
(d) $-x^2 + 5x + 6$

Solution:

Let the zeroes be α and β respectively.
Therefore, $\alpha + \beta = -5$ and $\alpha\beta = 6$.
Hence, the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta$$
$$= x^2 - (-5)x + 6$$
$$= x^2 + 5x + 6$$

Hence, the correct answer is option A.

Question: 8

The value of p, for which the points A(3, 1), B(5, p) and C(7, −5) are collinear, is
(a) −2
(b) 2
(c) −1
(d) 1

Solution:

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Given $A(3,\ 1),\ \ B(5,\ p)$ and $C(7,\ \ -5)$ are collinear.

$\Rightarrow \text{Area of } \Delta\,\text{ABC},\ A=0$

$\Rightarrow \frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right]=0$

$\Rightarrow \left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right]=0$

$\Rightarrow \left[3\left(p+5\right)+5\left(-5-1\right)+7\left(1-p\right)\right]=0$

$\Rightarrow \left[3p+15-30+7-7p\right]=0$

$\Rightarrow -4p-8=0$

$\Rightarrow 4p=-8$

$\Rightarrow p=-2$

Hence, the correct answer is option A.

Question: 9

The distance between the points (a cos θ + b sin θ, 0) and (0, a sin θ − b cos θ), is

(a) a^2+b^2

(b) a^2-b^2

(c) $\sqrt{a^2+b^2}$

(d) $\sqrt{a^2-b^2}$

Solution:

The distance between two points $\text{P}\left(x_1, y_1\right)$ and $\text{Q}\left(x_2, y_2\right)$ is given by $\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$

Thus, the distance between the two given points is given by

$=\sqrt{\left[0-\left(a\cos\theta+b\sin\theta\right)\right]^2+\left[\left(a\sin\theta-b\cos\theta\right)-0\right]^2}$

$=\sqrt{\left(a\cos\theta+b\sin\theta\right)^2+\left(a\text{sin}\theta-b\text{cos}\theta\right)^2}$

$=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta+2ab\sin\theta\cos\theta+a^2\sin^2\theta+b^2\cos^2\theta-2ab\sin\theta\cos\theta}$

$=\sqrt{a^2\left(\cos^2\theta+\sin^2\theta\right)+b^2\left(\sin^2\theta+\cos^2\theta\right)}$

$=\sqrt{a^2\times 1+b^2\times 1}$

$=\sqrt{a^2+b^2}$

Hence, the correct answer is option (c).

Question: 10

If the point P(k, 0) divides the line segment joining the points A(2, −2) and B(−7, 4) in the ratio 1 : 2, then the value of k is

(a) 1

(b) 2

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(c) −2
(d) −1

Solution:

Using the Section Formula, we have

$k=\frac{1\times(-7)+2\times 2}{1+2}$

$\Rightarrow \quad k=\frac{-7+4}{3}$

$\Rightarrow \quad k=\frac{-3}{3}$

$\Rightarrow \quad k=-1$

Hence, the correct answer is option D.

Question: 11

Fill in the blank.

Given ΔABC ~ ΔPQR, if $\frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{1}{3}$, then $\frac{\mathrm{ar}(\Delta \mathrm{ABC})}{\mathrm{ar}(\Delta \mathrm{PQR})}=$ __________.

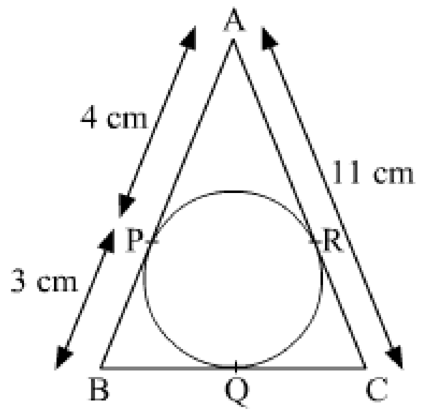
Solution:

Given that △ABC ~ △PQR and $\frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{1}{3}$.

We know that $\frac{\mathrm{ar}(\triangle \mathrm{ABC})}{\mathrm{ar}(\triangle \mathrm{PQR})}=\left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right)^2=\left(\frac{1}{3}\right)^2=\frac{1}{9}$.

Question: 12

Fill in the blank.
In the given figure ΔABC is circumscribing a circle, the length of BC is _____ cm.



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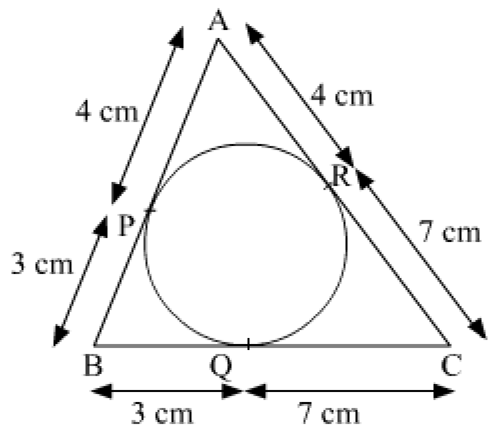
Solution:



Since we know that the lengths of tangents drawn from an exterior point to a circle are equal.

Therefore, AP = AR = 4 cm, BP = BQ = 3 cm.
Therefore, CR = AC − AR = $11 - 4 = 7$ cm.
Hence, BC = BQ + CQ = BQ + CR = $3 + 7$ cm = 10 cm.

Question: 13

Fill in the blank.
The value of $\left(\sin^2\theta + \frac{1}{1+\tan^2\theta}\right)$ = ___________.

OR

Fill in the blank.
The value of (1 + $\tan^2$ θ) (1 − sin θ) (1 + sin θ) = ____________.

Solution:

$$\sin^2\theta + \frac{1}{1+\tan^2\theta}$$
$$= \sin^2\theta + \frac{1}{\sec^2\theta} = \sin^2\theta + \cos^2\theta = 1$$

OR

The given expression is
$$(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$$
$$= (1 + \tan^2\theta)(1 - \sin^2\theta)$$
$$= \sec^2\theta . \cos^2\theta$$
$$= \frac{1}{\cos^2\theta} . \cos^2\theta$$
$$= 1$$
Thus, the value of (1 + $\tan^2$ θ) (1 − sin θ) (1 + sin θ) is 1.

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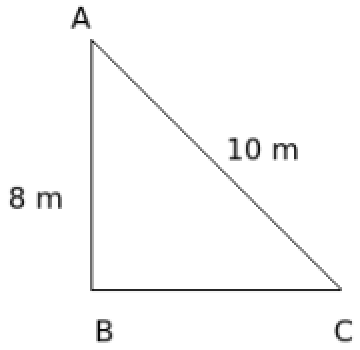
Question: 14

Fill in the blank.

A ladder 10 m long reaches a window 8 m above the ground. The distance of the foot of the ladder from the base of the wall is ________ m.

Solution:



Let AC be the ladder and AB be the wall.

In ΔABC, $AC^2 = AB^2 + BC^2$ [Pythagoras Theorem]

$\Rightarrow (10)^2 = (8)^2 + BC^2$

$\Rightarrow BC^2 = 100 - 64 = 36$

$\Rightarrow BC = 6 \text{ m}$

Hence, foot of the ladder is 6 m away from the base of the wall.

Question: 15

Fill in the blank.

$\frac{2\cos 67^\circ}{\sin 23^\circ} - \frac{\tan 40^\circ}{\cot 50^\circ} - \cos 0^\circ =$ ________.

Solution:

$$\frac{2\cos 67^\circ}{\sin 23^\circ} - \frac{\tan 40^\circ}{\cot 50^\circ} - \cos 0^\circ$$
$$= \frac{2\cos(90^\circ - 23^\circ)}{\sin 23^\circ} - \frac{\tan(90^\circ - 50^\circ)}{\cot 50^\circ} - \cos 0^\circ$$
$$= \frac{2\sin 23^\circ}{\sin 23^\circ} - \frac{\cot 50^\circ}{\cot 50^\circ} - \cos 0^\circ$$
$$= 2 \times 1 - 1 - 1$$
$$= 2 - 1 - 1 = 0$$

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Question: 16

If the mean of the first *n* natural number is 15, then find *n*.

Solution:

Given: mean of the first *n* natural numbers is 15.

$\therefore \frac{1+2+3+...+n}{n} = 15$

$\Rightarrow 1 + 2 + 3 + \ldots + n = 15n$

$\Rightarrow \frac{n(n+1)}{2} = 15n$

$\Rightarrow n^2 + n = 30n$

$\Rightarrow n^2 - 29n = 0$

$\Rightarrow n\left(n - 29\right) = 0$

$\Rightarrow n = 0,\ 29$

So, *n* = 29 (Since *n* cannot be zero)

Question: 17

A die is thrown once. What is the probability of getting a number less than 3?

OR

If the probability of winning a game is 0.07, what is the probability of losing it?

Solution:

When a die is thrown, all the outcomes are = {1, 2, 3, 4, 5, 6}
Total number of outcomes = 6
Favourable outcomes = {1, 2}
Favourable number of outcomes = 2

P(a number less than 3) = $\frac{2}{6} = \frac{1}{3}$

OR

P(winning) = 0.07
P(losing) = 1 — P(winning)
P(losing) = 1 — 0.07 = 0.93

Question: 18

The ratio of the length of a vertical rod and the length of its shadow is 1: √3. Find the angle of elevation of the sun at that moment?

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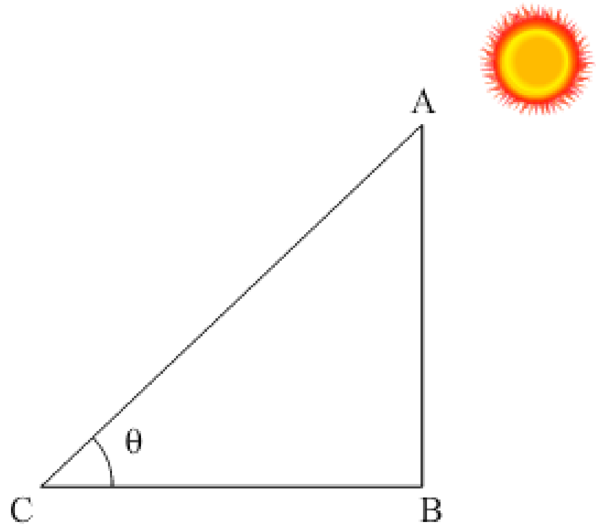
Solution:

Given that $\frac{AB}{BC} = \frac{1}{\sqrt{3}}$



From the figure, it is clear that △△ABC is a right-angled triangle in which AB is the vertical rod and BC is its shadow.

We have,

$$\tan\theta = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \tan\theta = \tan 30^{\circ}$$
$$\Rightarrow \theta = 30^{\circ}$$

Hence, the required angle of elevation of the sun is 30°.

Question: 19

Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

Solution:

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Let the heights, radii and volumes of the two cones be $(h_1,\ r_1,\ V_1)$ and $(h_2,\ r_2,\ V_2)$.
Given: $\frac{h_1}{h_2}=\frac{1}{3}$ and $\frac{r_1}{r_2}=\frac{3}{1}$

The required ratio of their volumes = $\frac{V_1}{V_2}$

$$=\frac{\frac{1}{3}\pi {r_1}^2 h_1}{\frac{1}{3}\pi {r_2}^2 h_2}$$

$$=\frac{{r_1}^2}{{r_2}^2}\times\frac{h_1}{h_2}$$

$$=\frac{3^2}{1}\times\frac{1}{3}$$

$$=\frac{3}{1}$$

$$=3:1$$

Hence, the required ratio of the volumes is 3 : 1.

Question: 20

A pair of dice is thrown once. What is the probability of getting a doublet?

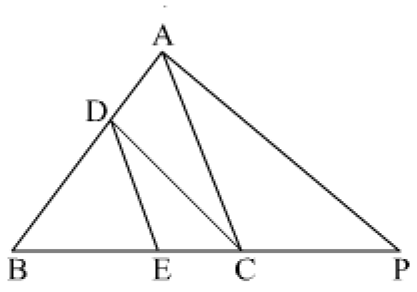
Solution:

The favourable outcomes for this event to happen are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) Favourable outcomes = 6

$$\text{Total no. of outcomes are} = 36$$
$$\text{So, P(doublet)} = \frac{6}{36} = \frac{1}{6}$$

Question: 21

In the given Figure, DE || AC and DC || AP. Prove that $\frac{\mathrm{BE}}{\mathrm{EC}}=\frac{\mathrm{BC}}{\mathrm{CP}}$



OR

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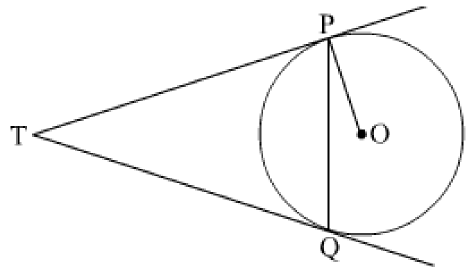
In the given Figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2 ∠OPQ.



Solution:

In ΔABP, DC || AP

By Basic Proportionality theorem,

$\frac{BD}{DA} = \frac{BC}{CP}$ .....(i)

In ΔBAC, DE || AC

By Basic Proportionality theorem,

$\frac{BD}{DA} = \frac{BE}{EC}$ .....(ii)

Thus, from (i) and (ii) we have

$\frac{BE}{EC} = \frac{BC}{CP}$

Hence proved.

Hence proved.

OR

Given: PT and TQ are the tangents to the circle with centre O.

To prove: $\angle PTQ = 2\angle OPQ$

Proof:

In $\triangle$PTQ,

PT = PQ (Tangents from an external point to the circle are equal)

$\Rightarrow \angle TPQ = \angle TQP$ (Angles opposite to equal sides are equal)

Let $\angle PTQ = \theta$

So, in $\triangle$PTQ

$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$

$\Rightarrow \angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{1}{2}\theta$

We know, angle made by the tangent with the radius is 90°.

So, $\angle OPT = 90^\circ$

Now,

$\angle OPT = \angle OPQ + \angle TPQ$

$\Rightarrow 90^\circ = \angle OPQ + \left(90^\circ - \frac{1}{2}\theta\right)$

$\Rightarrow \angle OPQ = \frac{1}{2}\theta = \frac{1}{2}\angle PTQ$

$\Rightarrow \angle PTQ = 2\angle OPQ$

Hence Proved.

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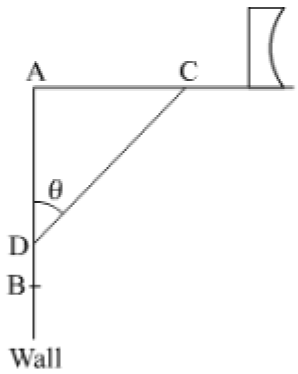
Question: 22

The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in the given figure. If AC = 1.5 m long and CD = 3 m, find (i) tanθ (ii) secθ + cosecθ



Solution:

In $\triangle$ACD, we have
AC = 1.5 cm, CD = 3 cm.

Since $\triangle$ACD is a right-angled triangle, so using Pythagoras Theorem, we have

$$\begin{aligned} AD^2 &= CD^2 - AC^2 \\ &= 3^2 - 1.5^2 \\ &= 6.75 \end{aligned}$$
$$\therefore AD = \sqrt{6.75} = 2.5 \text{ cm}$$

Consider
(i) $\tan\theta = \frac{AC}{AD} = \frac{1.5}{2.5} = \frac{3}{5}$
(ii) $\sec\theta + \operatorname{cosec}\theta = \frac{CD}{AD} + \frac{CD}{AC} = \frac{3}{2.5} + \frac{3}{1.5} = \frac{6}{5} + 2 = \frac{16}{5}$

Question: 23

If a number x is chosen at random from the numbers −3, −2, −1, 0, 1, 2, 3. What is probability that $x^2 \leq 4$?

Solution:

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The given numbers are $-3, -2, -1, 0, 1, 2, 3$.
Total number of possible outcomes = 7
Now, the favorable outcomes are given by $x^2 \leq 4$
i.e. $-2 \leq x \leq 2$
i.e. $-2, -1, 0, 1, 2$
Total number of favorable outcomes = 5
Hence, the required probability = $\frac{5}{7}$.

Question: 24

Find the mean of the following distribution:

Class:	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency:	5	10	10	7	8

OR

Find the mode of the following data :

Class:	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140
Frequency:	6	8	10	12	6	5	3

Solution:

Class	Frequency(f_i)	Class Mark(x_i)	$f_i x_i$
3 - 5	5	4	20
5 - 7	10	6	60
7 - 9	10	8	80
9 - 11	7	10	70
11 - 13	8	12	96
	$\sum f_i = 40$		$\sum f_i x_i$ = 326

Mean, $\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{326}{40} = 8.15$
Thus, mean = 8.15

OR

In the given data, the maximum class frequency is 12.
The class corresponding to the given class is 60 - 80, which is the modal class.

We have

Lower limit of modal class, l = 60
Frequency of modal class, f_1 = 12

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Frequency of a class preceding to modal class, f_o = 10
Frequency of a class succeeding to modal class, f_2 = 6
Class size h = 20

Mode = $l+\frac{(f_1-f_0)}{(2f_1-f_o-f_2)}\times h$
$=60+\frac{(12-10)}{(24-10-6)}\times 20$
$=60+\frac{2}{8}\times 20$
$=60+5$
$=65$
Hence, the mode of the given data is 65.

Question: 25

The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes.

Solution:

Length of minute hand of clock, $r=12$ cm
Angle described by the minute hand in 60 minutes $=360^{\circ}$
Angle described by the minute hand in 35 minutes, $\theta=\frac{360^{\circ}}{60}\times 35=210^{\circ}$
Area of a sector $=\frac{\theta}{360^{\circ}}\left(\pi r^2\right)=\frac{210^{\circ}}{360^{\circ}}\times\frac{22}{7}\times 12\times 12=264\ \text{cm}^2$

Question: 26

The sum of the first 7 terms of an AP is 63 and that of its next 7 terms is 161. Find the AP.

Solution:

Let a and d be the first term and common difference respectively.
Sum of the first 7 terms is given by, $S_7=\frac{7}{2}(2a+6d)=63$
$\Rightarrow a+3d=9$ .....(1)
Sum of the next 7 terms$=S_{14}-S_7=161$
$\Rightarrow \frac{14}{2}(2a+13d)-\frac{7}{2}(2a+6d)=161$
$\Rightarrow 7(2a+13d)-63=161$

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$\Rightarrow 14a + 91d = 224$ .....(2)

Multiplying both side of (1) by 14 and then subtracting from (2), we get

$49d = 98$

$\Rightarrow d = 2$

Putting $d = 2$ in (2), we get

$a = 3$

Therefore, the required AP is 3, 5, 7, 9, ...

Question: 27

Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

OR

If 4 is a zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes.

Solution:

The first given equation is $2y - x = 8$

x	0	−8
y	4	0

The second given equation is $5y - x = 14$

x	0	−14
y	2.8	0

The third given equation is $y - 2x = 1$

x	0	−0.5
y	1	0

Plotting the three given lines on the graph paper we get

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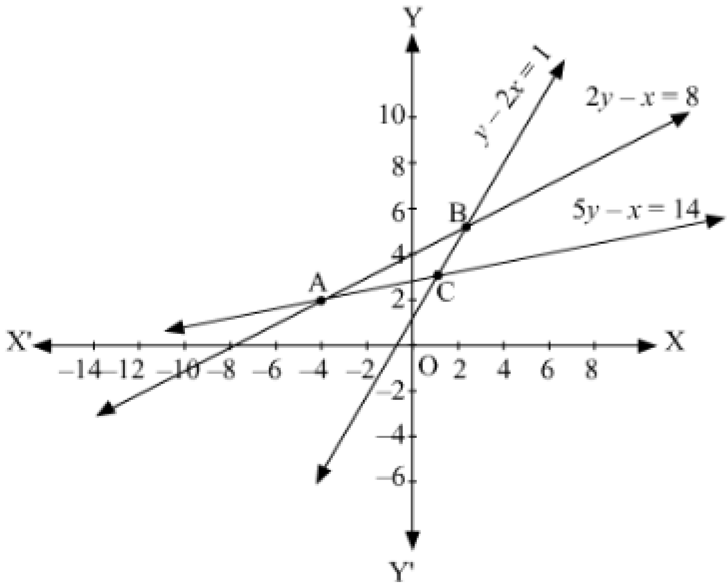

The coordinates of the vertices of the triangle ABC are A(−-4, 2), B(2, 5) and C(1, 3).

OR

Given 4 is a zero of a cubic polynomial $x^3 - 3x^2 - 10x + 24$

$\Rightarrow (x-4)$ is the factor of polynomial $x^3 - 3x^2 - 10x + 24$

Therefore, we have

$$
\begin{array}{r|l}
 & x^2 + x - 6 \\
x - 4 & x^3 - 3x^2 - 10x + 24 \\
 & x^3 - 4x^2 \\
 & - \quad + \\
\hline
 & x^2 - 10x + 24 \\
 & x^2 - 4x \\
 & - \quad + \\
\hline
 & -6x + 24 \\
 & -6x + 24 \\
 & + \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

To find the other two zeroes of the given polynomial, we need to find the zeroes of the quotient $x^2 + x - 6$.

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i.e. $x^2+x-6=0$
$\Rightarrow x^2+3x-2x-6=0$
$\Rightarrow x(x+3)-2(x+3)=0$
$\Rightarrow (x+3)(x-2)=0$
$\Rightarrow x+3=0$ or $x-2=0$
$\Rightarrow x=-3$ or $x=2$
Hence, the other two zeroes of the given polynomial are 2 and -3.

Question: 28

Find the area of triangle PQR formed by the points P(−5, 7), Q(−4, −5) and R (4, 5).

OR

If the point C(−1, 2) divides internally the line segment joining A(2, 5) and B(*x*, *y*) in the ratio 3 : 4, find the coordinates of B.

Solution:

Given: Vertices of the triangle are $\mathrm{P}(-5,7),\ \mathrm{Q}(-4,-5)$ and $\mathrm{R}(4,5)$.
Let A be the area of the triangle.
Using the formula to calculate the area of the triangle, we have

$$A=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$
$$=\frac{1}{2}\left[(-5)(-5-5)+(-4)(5-7)+(4)(7+5)\right]$$
$$=\frac{1}{2}\left[50+8+48\right]$$
$$=53$$

Hence, the area of the triangle is 53 square units.

OR

Since the point $\mathrm{C}(-1,\ 2)$ divides the line segment joining $\mathrm{A}(2,\ 5)$ and $\mathrm{B}(x,\ y)$ in the ratio $3:4$.
Therefore using the section-formula of internal division, we get
For x- coordinate,

$$-1=\frac{3\times x+4\times 2}{3+4}$$

$\Rightarrow 3x+8=-7$
$\Rightarrow x=-5$

For y- coordinate,

$$2=\frac{3\times y+4\times 5}{3+4}$$

$\Rightarrow 3y+20=14$
$\Rightarrow y=-2$
Hence, the coordinates of B are $(-5,\ -2)$.

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Question: 29

Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

OR

Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.

Solution:

The given quadratic polynomial is

$f(x) = ax^2 + bx + c,\ a \neq 0,\ c \neq 0$

Let α and β be the two zeroes of the given quadratic polynomial.

Then,

Sum of zeroes = $\alpha + \beta = \frac{-b}{a}$

Product of zeroes = $\alpha\beta = \frac{c}{a}$

$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c}$ and $\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{c}{a}} = \frac{a}{c}$

So, the required new quadratic polynomial is

$k\left[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}\right]$

$= k\left[x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha} \cdot \frac{1}{\beta}\right]$

$= k\left[x^2 - \left(\frac{-b}{c}\right)x + \frac{a}{c}\right]$

where k is a real number.

OR

Given,

$f(x) = 3x^2 - x^3 - 3x + 5$

$g(x) = x - 1 - x^2$

$$\begin{array}{r|l} & x - 2 \\ -x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\ & -x^3 + x^2 - x \\ & + \quad - \quad + \\ \hline & 2x^2 - 2x + 5 \\ & 2x^2 - 2x + 2 \\ & - \quad + \quad - \\ \hline & 3 \end{array}$$

So,

$q(x) = (x - 2)$ and $r(x) = 3$

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To verify: $f(x) = g(x) . q(x) + r(x)$

Verification:

$$g(x) . q(x) + r(x) = (-x^2 + x - 1)(x - 2) + 3$$
$$= -x^2(x - 2) + x(x - 2) - 1(x - 2) + 3$$
$$= -x^3 + 2x^2 + x^2 - 2x - x + 2 + 3$$
$$= -x^3 + 3x^2 - 3x + 5$$
$$= f(x)$$

Hence verified.

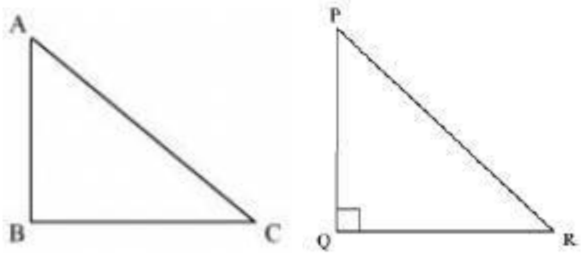
Question: 30

In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle.

Solution:

Given: In ΔABC, $AC^2 = AB^2 + BC^2$
To prove: ∠B = 90°

Construction: ΔPQR right-angled at Q such that PQ = AB and QR = BC



In ΔPQR,
$PR^2 = PQ^2 + QR^2$ (By Pythagoras Theorem, as ∠Q = 90°)

⇒ $PR^2 = AB^2 + BC^2$ ..... (1) (By construction)
However, $AC^2 = AB^2 + BC^2$ ..... (2) (Given)

From (1) and (2), we obtain
AC = PR ..... (3)

Now, In ΔABC and ΔPQR, we obtain
AB = PQ (By construction)
BC = QR (By construction)
AC = PR [From (3)]

Therefore, ΔABC ≅ ΔPQR (by SSS congruency criterion)
⇒ ∠B = ∠Q (By CPCT)

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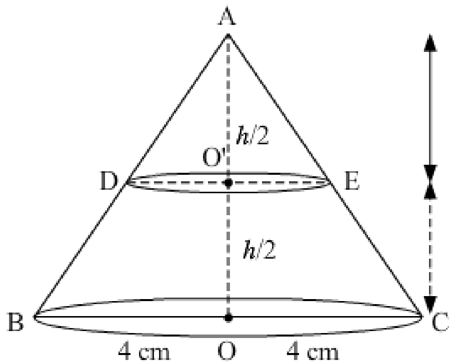
However, ∠Q = 90° (By construction)

∴ ∠B = 90°
Hence proved.

Question: 31

A cone of base radius 4 cm is divided into two parts by drawing a plane through the mid-points of its height and parallel to its base. Compare the volume of the two parts.

Solution:



Given: $\mathrm{OC}=4\,\mathrm{cm}, \quad \mathrm{AO'}=\mathrm{OO'}$
Let $\mathrm{AO}=h$
$\Rightarrow \mathrm{AO'}=\mathrm{OO'}=\frac{h}{2}$

In $\triangle \mathrm{AO'C}$ and $\triangle \mathrm{AOC}$
$\angle \mathrm{E}=\angle \mathrm{C}$ [Corresponding angles]
$\angle \mathrm{A}=\angle \mathrm{A}$ [Common angle]
$\Rightarrow \triangle \mathrm{AO'E} \cong \triangle \mathrm{AOC}$ [By SS similarity criterion]

Therefore, $\frac{\mathrm{O'E}}{\mathrm{OC}}=\frac{\mathrm{AO'}}{\mathrm{AO}}=\frac{1}{2}$
$\Rightarrow \frac{\mathrm{O'E}}{\mathrm{OC}}=\frac{1}{2}$
Let $V_1,\ V_2$ are the volumes of the cone ADE and cone ABC respectively.

$$\frac{V_1}{V_2}=\frac{\left[\frac{1}{3}\pi(\mathrm{O'E})^2\,\mathrm{AO'}\right]}{\left[\frac{1}{3}\pi(\mathrm{OC})^2\,\mathrm{AO}\right]}$$

$$=\left(\frac{\mathrm{O'E}}{\mathrm{OC}}\right)^2\left(\frac{\mathrm{AO'}}{\mathrm{AO}}\right)$$

$$=\frac{1}{4}\times\frac{1}{2}$$

$$=\frac{1}{8}$$

$$\frac{\text{Volume of the upper part of the cone}}{\text{Volume of the lower part of the cone}}=\frac{V_1}{V_2-V_1}$$

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$$= \frac{\left(\frac{V_1}{V_2}\right)}{1-\left(\frac{V_1}{V_2}\right)}$$

$$= \frac{1}{7} \qquad \left(\because \frac{V_1}{V_2} = \frac{1}{8}\right)$$

Question: 32

A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream.

Solution:

Let x and y be the speed of boat in still water and stream respectively.
Therefore,
Speed of the boat in downstream$= (x + y)$ km/h
Speed of the boat in upstream$= (x - y)$ km/h

Applying the first Condition 1, which is: A man can row a boat downstream 20 km in 2 hours.

$x + y = \frac{20}{2}$

$\Rightarrow\ x + y = 10$ .....(1)

Condition 2: A man can row a boat upstream 4 km in 2 hours.

$x - y = \frac{4}{2}$

$\Rightarrow\ x - y = 2$ .....(2)

Adding (1) and (2), we get
$2x = 12 \Rightarrow x = 6$ km/h

substituting $x = 6$ in (2)
$6 - y = 2 \Rightarrow y = 4$ km/h

Hence, the speed of rowing in still water = 6 km/h.
The speed of the stream = 4 km/h.

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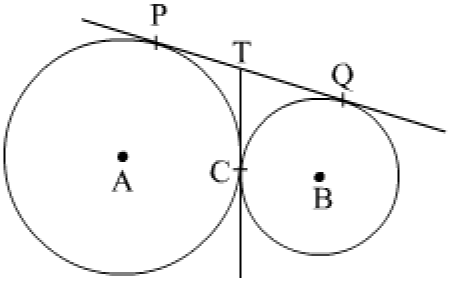
Question: 33

In the given figure 5, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



Solution:

PT and TC are the tangents to the circle with centre A from an external point T.
Therefore, PT = TC .....(i) (Tangents to a circle drawn from an external point are equal)

TC and TQ are the tangents to the circle with centre B from an external point T.
Therefore, TC = TQ .....(ii) (Tangents to a circle drawn from an external point are equal)

From (i) and (ii), we get
PT=TQ

Hence proved.

Question: 34

Prove that : $\frac{\cot\ \theta+\text{cosec}\ \theta-1}{\cot\ \theta-\text{cosec}\ \theta+1}=\frac{1+\cos\ \theta}{\sin\ \theta}$

Solution:

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$$\frac{\cot\ \theta+\text{cosec}\ \theta-1}{\cot\ \theta-\text{cosec}\ \theta+1}$$

$$=\frac{\cot\ \theta+\text{cosec}\ \theta-(\text{cosec}^2\ \theta-\cot^2\ \theta)}{\cot\ \theta-\text{cosec}\ \theta+1} \quad [\because\ \text{cosec}^2\ \theta=1+\cot^2\ \theta]$$

$$=\frac{\cot\ \theta+\text{cosec}\ \theta-(\text{cosec}\ \theta-\cot\ \theta)(\text{cosec}\ \theta+\cot\ \theta)}{\cot\ \theta-\text{cosec}\ \theta+1}$$

$$=\frac{(\text{cosec}\ \theta+\cot\ \theta)(1-\text{cosec}\ \theta+\cot\ \theta)}{\cot\ \theta-\text{cosec}\ \theta+1}$$

$$=\text{cosec}\ \theta+\cot\ \theta$$

$$=\frac{1}{\sin\theta}+\frac{\cos\ \theta}{\sin\theta}$$

$$=\frac{1+\cos\ \theta}{\sin\ \theta}$$

Hence proved.

Question: 35

The following table gives production yield per hectare (in quintals) of wheat of 100 farms of a village :

Production yield/hect.	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70
No. of farms	4	6	16	20	30	24

Change the distribution to 'a more than' type distribution and draw its ogive.

OR

The median of the following data is 525. Find the values of *x* and *y*, if total frequency is 100:

Class :	**Frequency:**
0 – 100	2
100 – 200	5
200 – 300	*x*
300 – 400	12
400 – 500	17
500 – 600	20
600 – 700	*y*
700 – 800	9
800 – 900	7
900 – 1000	4

Solution:

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Given:

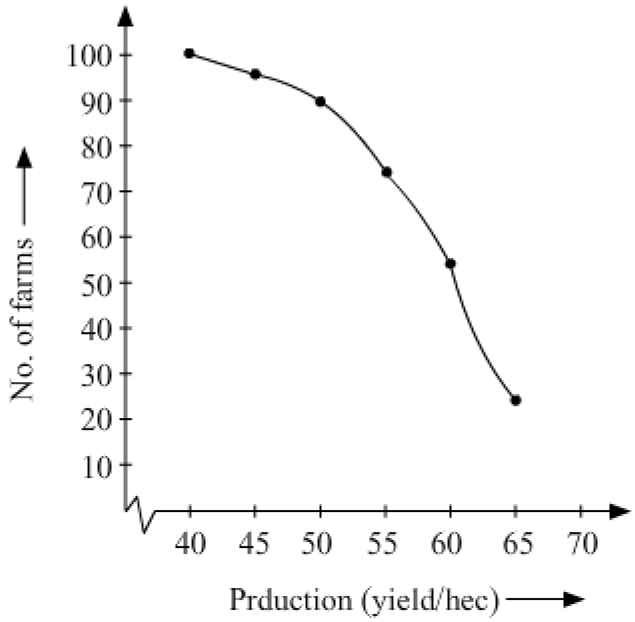
Production yield/hect.	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70
No. of farms	4	6	16	20	30	24

"more than type" distribution table is as follows-

Production (yield/hec)	No. of farms
More than 40	100
More than 45	96
More than 50	90
More than 55	74
More than 60	54
More than 65	24

To draw the ogive, we have the following points-
(40, 100), (45, 96), (50, 90), (55, 74), (60, 54), (65, 24)

Plotting these points, we get the following ogive-



OR

Given, median = 525

We prepare the cumulative frequency table, as given below.

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Class interval:	Frequency: (f_i)	Cumulative frequency $(c.f.)$
0-100	2	2
100-200	5	7
200-300	f_1	$7+f_1$
300-400	12	$19+f_1$
400-500	17	$36+f_1$
500-600	20	$56+f_1$
600-700	f_2	$56+f_1+f_2$
700-800	9	$65+f_1+f_2$
800-900	7	$72+f_1+f_2$
900−1000	4	$76+f_1+f_2$
	$N=100=76+f_1+f_2$	

$$N=100$$
$$76+f_1+f_2=100$$
$$f_2=24-f_1 \quad .....(1)$$
$$\frac{N}{2}=50$$

Since median = 525,
So, the median class is $500-600$.

Here, $l=500, f=20, F=36+f_1$ and $h=100$

We know that

$$\text{Median}=l+\left\{\frac{\frac{N}{2}-F}{f}\right\}\times h$$

$$525=500+\left\{\frac{50-(36+f_1)}{20}\right\}\times 100$$

$$25=\frac{(14-f_1)\times 100}{20}$$

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$$25 \times 20 = 1400 - 100 f_1$$
$$100 f_1 = 1400 - 500$$
$$f_1 = \frac{900}{100}$$
$$= 9$$

Putting the value of f_1 in (1), we get

$$f_2 = 24 - 9$$
$$= 15$$

Hence, the missing frequencies are 9 and 15.

Question: 36

A bucket in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm, respectively. Find the capacity of the bucket. Also find the cost of milk which can completely fill the bucket at the rate of Rs. 40 per litre.
$\left(\text{Use } \pi = \frac{22}{7}\right)$

Solution:

It is given that radius of upper end of the bucket = r_1 = 20 cm
Radius of lower end of the bucket = r_2 = 10 cm

Height of the bucket, h = 30 cm
The bucket is in the shape of frustum of a cone.

Therefore, volume of the bucket

$$= \frac{\pi}{3} h \left({r^2}_1 + {r^2}_2 + r_1 r_2\right)$$
$$= \frac{22}{7 \times 3} \times 30 \left(20^2 + 10^2 + 20 \times 10\right)$$
$$= \frac{220}{7} (700)$$
$$= 22000 \text{ cm}^3$$

$\text{Amount of milk the bucket can hold} = \frac{22000}{1000} = 22 \text{ L}$

$\text{Total cost of milk} = 22 \times 40 = 880 \text{ Rs}$

Question: 37

Show that the square of any positive integer cannot be of the form ($5q$ + 2) or ($5q$ + 3) for any integer q.

OR

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Prove that one of every three consecutive positive integers is divisible by 3.

Solution:

Let *b* be an arbitrary positive integer.
By Euclid's division lemma,
$b = aq + r$, where $0 \leq r < a$

Now, if we divide *b* by 5, then *b* can be written in the form of $5m$, $5m+1$, $5m+2$, $5m+3$ or $5m+4$.
This implies that we have five possible cases.

Case I:
If $b = 5m$

Squaring both sides, we get

$$b^2 = (5m)^2 = 25m^2 = 5\left(5m^2\right)$$
$$\Rightarrow b^2 = 5q$$
where $q = 5m^2$ is an integer.

Case II:
If $b = 5m + 1$,
Squaring both sides, we get
$$b^2 = (5m+1)^2 = 25m^2 + 1 + 10m$$
$$\Rightarrow b^2 = 5\left(5m^2 + 2m\right) + 1$$
$$\Rightarrow b^2 = 5q + 1$$
where $q = 5m^2 + 2m$ is an integer.

Case III:
If $b = 5m + 2$
Squaring both sides, we get
$$b^2 = (5m+2)^2 = 25m^2 + 4 + 20m$$
$$\Rightarrow b^2 = 5\left(5m^2 + 4m\right) + 4$$
$$\Rightarrow b^2 = 5q + 4$$

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where $q = 5m^2 + 4m$ is an integer.

Case IV:
If $b = 5m + 3$
Squaring both sides, we get
$$b^2 = (5m+3)^2 = 25m^2 + 9 + 30m$$
$$\Rightarrow b^2 = 25m^2 + 5 + 4 + 30m$$
$$\Rightarrow b^2 = 5\left(5m^2 + 1 + 6m\right) + 4$$
$$\Rightarrow b^2 = 5q + 4$$
where $q = 5m^2 + 1 + 6m$ is an integer.

Case V:
If $b = 5m + 4$
Squaring both sides, we get
$$b^2 = (5m+4)^2 = 25m^2 + 16 + 40m$$
$$\Rightarrow b^2 = 25m^2 + 15 + 1 + 40m$$
$$\Rightarrow b^2 = 5\left(5m^2 + 3 + 8m\right) + 1$$
$$\Rightarrow b^2 = 5q + 1$$
where $q = 5m^2 + 3 + 8m$ is an integer.

Hence, we can conclude that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer.

OR

Let n, $n + 1$, $n + 2$ be three consecutive positive integers, where n is any natural number.
By Euclid's division lemma,
$n = aq + r$, where $0 \le r < a$.

Now, if we divide n by 3, then n can be written in the form of $3q$, $3q+1$ or $3q+2$.
This implies that we have three possible cases.

Case I:
If $n = 3q$, then n is divisible by 3.
However, $n + 1$ and $n + 2$ are not divisible by 3.

Case II:
If $n = 3q + 1$, then $n + 2 = 3q + 3 = 3(q + 1)$, which is divisible by 3.
However, n and $n + 1$ are not divisible by 3.

Case III:
If $n = 3q + 2$, then $n + 1 = 3q + 3 = 3(q + 1)$, which is divisible by 3.
However, n and $n + 2$ are not divisible by 3.

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Hence, we conclude that one of any three consecutive positive integers must be divisible by 3.

Question: 38

The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7 : 15. Find the numbers.

OR

Solve : 1 + 4 + 7 + 10 + ... + x = 287

Solution:

Let the four terms of the AP be $a - 3d$, $a - d$, $a + d$ and $a + 3d$.

Given:

$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

$\Rightarrow 4a = 32$

$\Rightarrow a = 8$

Also,

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$
$$\Rightarrow \frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$$
$$\Rightarrow \frac{(8)^2-9d^2}{(8)^2-d^2} = \frac{7}{15}$$
$$\Rightarrow \frac{64-9d^2}{64-d^2} = \frac{7}{15}$$

$$\Rightarrow 960 - 135d^2 = 448 - 7d^2$$
$$\Rightarrow 512 = 128d^2$$
$$\Rightarrow d^2 = 4$$
$$\Rightarrow d = \pm 2$$

When $a = 8$ and $d = 2$, then the terms are 2, 6, 10, 14.
When $a = 8$ and $d = -2$, then the terms are 14, 10, 6, 2.

OR

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In the given AP, we have

$a = 1,\ d = 3,\ S_n = 287$

The formula for sum of n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$.

This implies

$\frac{n}{2}[2(1) + (n-1)(3)] = 287$

$\Rightarrow n(2 + 3n - 3) = 574$

$\Rightarrow 3n^2 - n - 574 = 0$

$\Rightarrow 3n^2 - 42n + 41n - 574 = 0$

$\Rightarrow 3n(n-14) + 41(n-14) = 0$

$\Rightarrow (n-14) = 0 \text{ or } 3n + 41 = 0$

$\Rightarrow n = 14 \text{ or } n = -\frac{41}{3}$

$\therefore\ n = 14$

$x = a_{14} = a + (14-1)d = 1 + 13(3) = 1 + 39 = 40$

Thus, the value of x is 40.

Question: 39

Draw a ΔABC with BC = 7 cm, ∠B = 45° and ∠A = 105°. Then construct another triangle whose sides are 3/4 times the corresponding sides of ΔABC.

Solution:

In order to construct ΔABC, we follow the following steps:

STEP I: Draw BC = 7 cm.
STEP II: At B, construct ∠CBX = 45° and at C, construct ∠BCY=180°−(45°+105°) = 30°.

Let them intersect at AA. ΔABC so obtained is the given triangle.

To construct a triangle similar to ΔABC, we follow the following steps.

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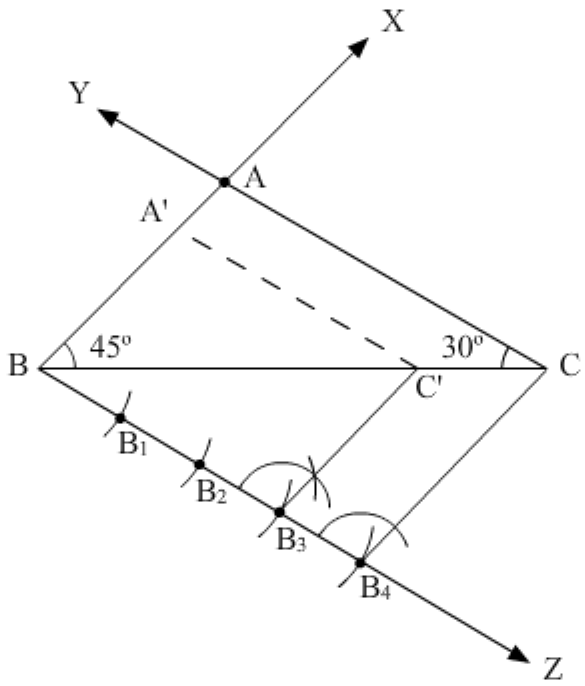

STEP I: Construct an acute angle $\angle CBZ$ at B on opposite side of vertex A of ΔABC.
STEP II: Mark-off four points $B_1,\ B_2,\ B_3,\ B_4$ on BZ such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
STEP III: Join B_4 to C and draw a line through B_3 parallel to B_4C, intersecting the line segment BC at C'.
STEP IV: Draw a line through C' parallel to CA intersecting the line segment BA at A'. Triangle $A'BC'$ so obtained is the required triangle such that $\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{3}{4}$.

Question: 40

From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Solution:

Let AB and EC be the building and tower respectively.

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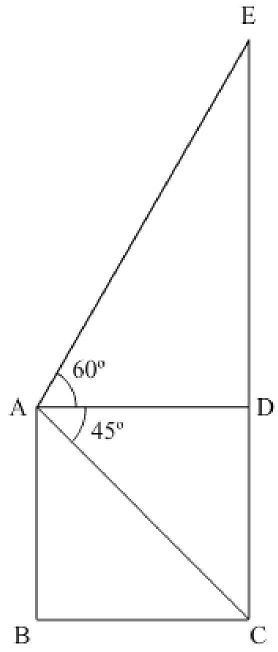

AB = 7 m
AB = CD = 7 m
In $\Delta \mathrm{ACD}, \angle \mathrm{A} = 45^{\circ}$
$\tan 45^{\circ} = \frac{\mathrm{CD}}{\mathrm{AD}}$
$\Rightarrow 1 = \frac{7}{\mathrm{AD}}$

$\mathrm{AD} = 7$ m .....(1)
In $\Delta \mathrm{AED}, \angle \mathrm{A} = 60^{\circ}$
$\tan 60^{\circ} = \frac{\mathrm{ED}}{\mathrm{AD}}$
$\Rightarrow \sqrt{3} = \frac{\mathrm{ED}}{7}$ [From (1)]
$\mathrm{ED} = 7\sqrt{3}$ m
Now, the length of the tower i.e. EC = ED $+$ DC
EC = $7\sqrt{3} + 7 = 7\left(\sqrt{3} + 1\right)$ m.

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